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1. Recognizing Quadratic Equations

A *quadratic equation* is an equation that can be put in the form

$$ax^2 + bx + c = 0,$$

where a, b, and c are numbers, and x is a variable.

Example 1. The equation $5x + 8x^2 = 3x + 7$ is a quadratic equation. Find a, b, and c.

Solution. First, we wish to put it in the *standard form* $ax^2 + bx + c = 0$.

$$5x + 8x^2 = 3x - 7$$

$$8x^2 + 5x = 3x - 7 \qquad \text{use the commutative property to switch } 5x \text{ and } 8x^2$$

$$8x^2 + 2x = -7 \qquad \text{subtract } 3x \text{ from both sides}$$

$$8x^2 + 2x + 7 = 0 \qquad \text{add 7 from both sides}$$

Now align this with the standard form. The number in front of x^2 is the a, the number in front of the x is the b, and the other number is the c. Thus $a = 8$, $b = 2$, and $c = 7$. □

Example 2. The equation $x^2 = 4$ is a quadratic equation. Find a, b, and c.

Solution. Subtract 4 from both sides to get $x^2 - 4 = 0$. This is the same as $1 \cdot x^2 + 0 \cdot x + (-4) = 0$. So, $a = 1$, $b = 0$, and $c = -4$. □

2. Solution Sets

For every *positive* real number c, there are two real numbers whose square is c. We define $\sqrt{c}$ to be the unique positive real number whose square is c; the other square root of c is negative, and is written $-\sqrt{c}$.

Example 3. Solve the quadratic equation $x^2 - 4 = 0$.

Solution. We add 4 to both sides to get $x^2 = 4$. When we take the square root of both sides, we write $x = \pm\sqrt{4}$ to indicate that there are two solutions. Since $\sqrt{4} = 2$, we have $x = \pm 2$. □

The *solution set* of an equation is the set of all solutions. Typically, it is understood that we are looking for real solutions, although there will be time you are asked to find all complex solutions; more on that later.

The solution set of a linear equation contains only one element. For example, the solution set of $x - 4 = 0$ is $\{4\}$.

Quadratic equations have either two, one, or zero solutions in the reals. For example, the solution set of $x^2 = 4$ is $\{-2, 2\}$.

We give some examples.

Equation	Solution set
$x - 4 = 7$	$\{11\}$
$7x + 10 = 0$	$\left\{-\dfrac{10}{7}\right\}$
$x^2 - 1 = 0$	$\{-1, 1\}$
$x^2 - 4 = 0$	$\{-2, 2\}$
$x^2 - 2 = 0$	$\{-\sqrt{2}, \sqrt{2}\}$
$x^2 - 2x + 1$	$\{1\}$
$x^2 - x = 6$	$\{-2, 3\}$
$x^3 - 7x + 6 = 0$	$\{1, 2, -3\}$

We may write the set$\{-2, 2\}$ as $\{\pm 2\}$.

3. Methods of Solving Quadratic Equations

We will learn four methods for solving quadratic equation. These are:

- Extracting roots
- Factoring
- Completing the square
- Quadratic formula

All depend on the first, which is extraction of roots.

4. Extraction of Roots

We manipulate the equation until it is of the form $x^2 = c$, where c is a real number, and then take the square root of both sides to get $x = \pm\sqrt{c}$.

Example 4. Solve $5x^2 - 25 = 0$.

Solution. When there is no x term (the b is zero), we put the x^2 stuff on the left side and the numeric stuff on the other side.

$$\begin{aligned} 5x^2 - 25 &= 0 \\ 5x^2 &= 25 && \text{add 25 from both sides} \\ x^2 &= 5 && \text{divide both sides by 5} \\ x &= \pm\sqrt{5} && \text{take the square root of both sides} \end{aligned}$$

The solution set is $\{\pm\sqrt{5}\}$. □

Example 5. Solve $3x^2 - 10 = x^2 + 8$.

Solution. Get x^2 on the left side and a number right side.

$$\begin{aligned} 3x^2 - 10 &= x^2 + 8 \\ 2x^2 - 10 &= 8 && \text{subtract } x^2 \text{ from both sides} \\ 2x^2 &= 18 && \text{add ten to both sides} \\ x^2 &= 9 && \text{divide both sides by 2} \\ x &= \pm 3 && \text{take the square root of both sides} \end{aligned}$$

The solution set is $\{-3, 3\}$. □

Example 6. Solve $x^2 + 49 = 2x^2$.

Solution. Get just x^2 on the left side and a number right side. Since we want the x^2 on the left, but there are more x^2's on the right, the first thing we do is switch the sides.

$$\begin{aligned} x^2 + 49 &= 2x^2 \\ 2x^2 &= x^2 + 49 && \text{use the symmetric property of equality to switch the sides} \\ x^2 &= 49 && \text{subtract } x^2 \text{ from both sides.} \\ x &= \pm 7 && \text{take the square root of both sides} \end{aligned}$$

The solution set is $\{\pm 7\}$. □

In the next lesson, we will discuss factoring quadratics.